Technology

How to figure true temperature difference in shell-and-tube exchangers

The charts shown here greatly simplify an often onerous chore

BY DALE L. GULLEY
Engineering Consultant, Tulsa, Okla.

WANT TO CALCULATE the true temperature difference in shell-and-tube exchangers? Here is a simplified method for doing it for any of the common flow patterns.

And here are new, previously unpublished correction - factor charts for divided-flow and split-flow patterns.

These charts enable the designer to liberalize his ratings. Previously, normal single-shell-pass charts were used, and the correction factor, F, was kept above about 0.9. With the new charts, designs will sometimes call for fewer shells, and therefore will be more economical.

The basic equation for designing heat exchangers is:

$$A = Q/U\,\Delta T$$

Where:

A = external surface of tubes, sq ft

Q = heat load, Btu/hour

U = overall heat-transfer coefficient, Btu/hour-sq ft-°F.

ΔT = mean temperature difference, °F.

The problem is to find the true ΔT.

The simplest temperature difference occurs when an exchanger has constant temperature on both sides: for instance, when steam condenses on one side and a pure organic boils on the other. With steam at 366° F. and the other fluid boiling at 266° F., the temperature difference would be

$$366 - 266 = 100° \text{ F.}$$

This would be the temperature difference used in Equation 1. This holds true no matter what the two fluid flow patterns are.

Pattern for countercurrent flow

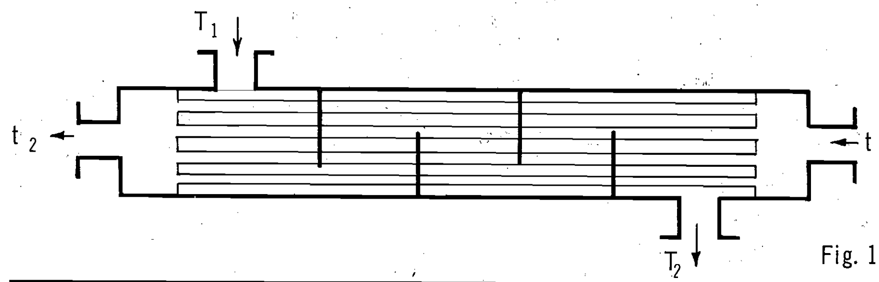


Fig. 1

1. Countercurrent flow

Fig. 1 is a simplified diagram of an exchanger in countercurrent flow. When there is sensible-heat transfer involved, this type of flow gives the most efficient temperature driving force and the biggest temperature cross, because the outlet temperature of the hot stream can be cooler than the outlet temperature of the cold stream. For example:

Hot	Cold
↓ 200	150 ↑
100	80

The ΔT for this flow pattern is defined by Equation 2:

$$\Delta T = \frac{GTD - LTD}{\ln \frac{GTD}{LTD}} \qquad (2)$$

Where:

GTD = greatest terminal temperature difference

LTD = least terminal temperature difference

ΔT = log mean temperature difference (LMTD)

Again referring to Fig. 1, the terminal temperature differences are $T_1 - t_2$ and $T_2 - t_1$. Assuming that $T_2 - t_1$ is the greater temperature difference:

$$\Delta T = \frac{(T_2 - t_1) - (T_1 - t_2)}{\ln \left(\frac{T_2 - t_1}{T_1 - t_2}\right)} \qquad (2a)$$

2. Cocurrent flow

Fig. 2 is a simplified diagram of an exchanger in cocurrent flow. Equation 2 is still useful, but the terminal temperature differences will

Pattern for cocurrent flow

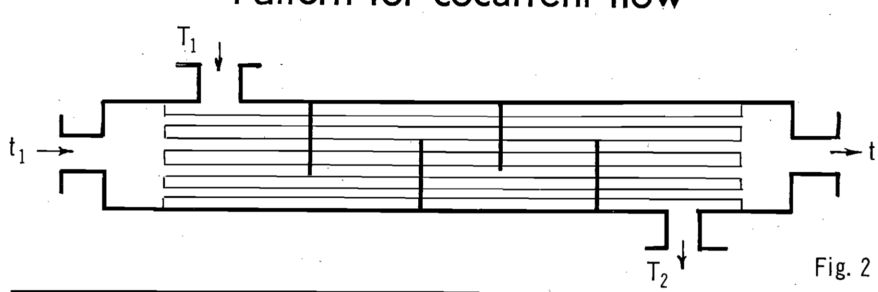


Fig. 2

be different. For example, one difference will be $T_1 - t_1$, and the other will be $T_2 - t_2$.

This type of flow pattern is seldom used. It is not very efficient and therefore will not cool a given fluid as much as the countercurrent flow will. The hot outlet temperature can only approach the cold outlet; it cannot cross it.

But use can be made of this inability to cross temperatures. For example, in wax and asphaltic coolers, cocurrent flow is used to make sure that the solidification point will not be reached. If countercurrent flow were used, there would be danger of cooling below design when the exchanger is clean.

3. Single-shell, multitube pass

In most heat exchangers, the flow is neither pure countercurrent nor cocurrent. Usually an exchanger will have multiple tube passes with the shell fluid sweeping back and forth across the outside of the tube bundle.

Fig. 3 is a simplified case of a multitube-pass exchanger. The shell fluid will be in countercurrent flow with one tube pass and cocurrent with the other.

It is possible to find the true overall temperature difference by a trial and error, but a much faster chart method is commonly used. This chart method is based on applying a correction factor to the log-mean-temperature difference. Then, the true temperature difference for this flow pattern will be:

$$\Delta T_c = \text{LMTD}\,(F) \qquad (3)$$

Where LMTD is defined by Equation 2

F = correction factor

If there is constant temperature on either side, F will be 1.0.

Fig. 4 is the LMTD correction factor for a one-shell pass, two-or-more tube-pass exchanger. Several publications[1 2 3] give correction-factor curves for one to six shells in series and even number of tube passes.

To use the correction curves it is necessary to calculate two dimensionless parameters. The parameter on the curves is called R and is equal to:

$$R = \frac{T_1 - T_2}{t_2 - t_1} \text{ or } \frac{wc}{WC} \qquad (4)$$

Where:

wc = heat capacity of tube fluid, Btu/°F.

WC = heat capacity of shell fluid, Btu/°F.

The variable on the abscissa is called X and is defined by:

$$X = \frac{t_2 - t_1}{T_2 - t_1} \qquad (5)$$

As shown in Fig. 4, at high values of R it is difficult to read F accurately. To overcome this problem, the parameters of R and X can be redefined:

Pattern for one-shell pass, two-tube pass

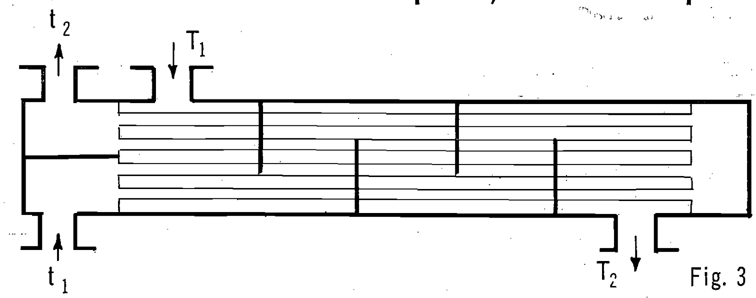


Fig. 3

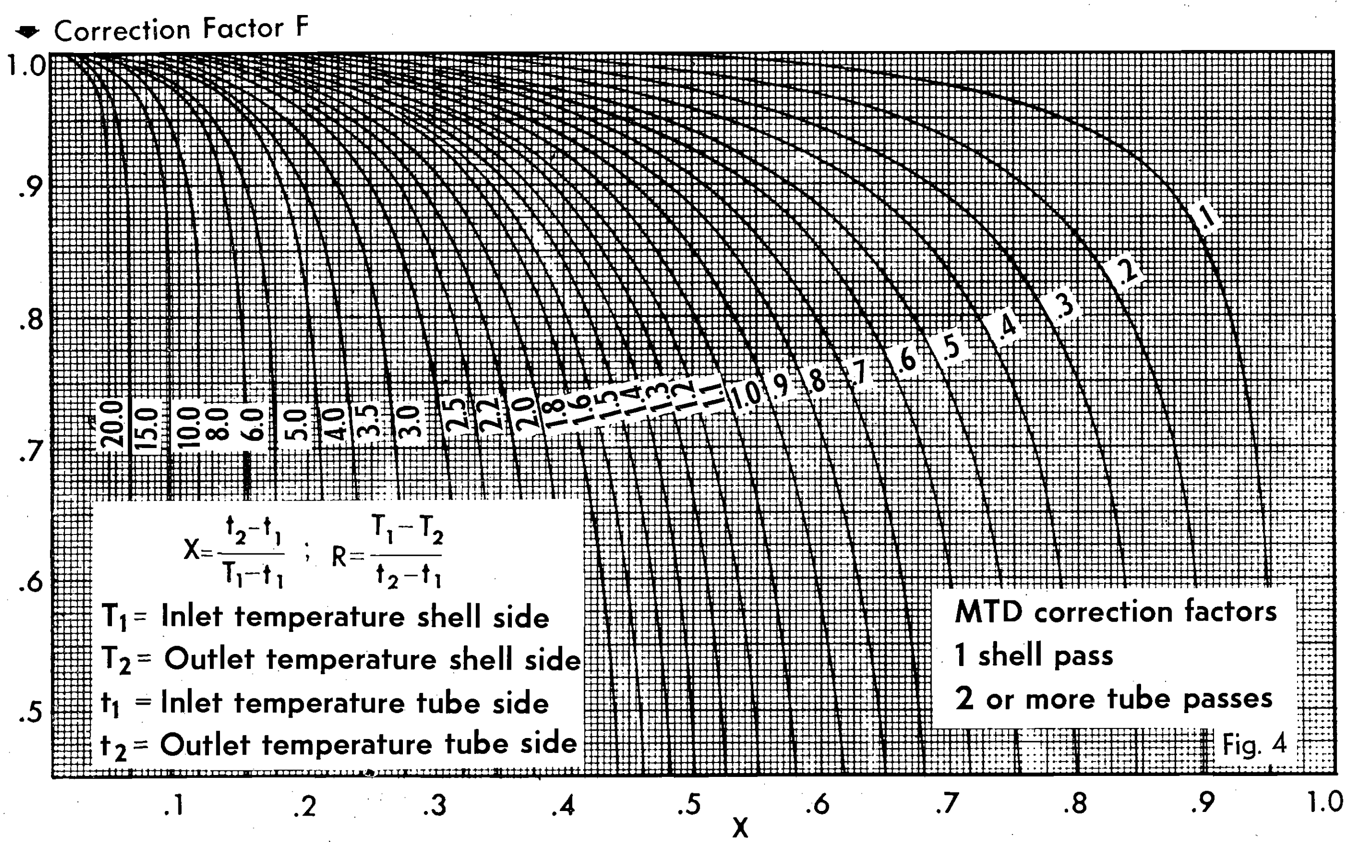


Fig. 4

$$R = \frac{\text{smallest temperature range}}{\text{greatest temperature range}} \quad (4a)$$

$$X = \frac{\text{greatest temperature range}}{T_1 - t_1} \quad (5a)$$

It follows that R will always be 1 or less, and that the curves will always have a relatively flat slope. For example, when the shell fluid cools from 190° to 90° and the tube-side fluid heats from 80° to 90°. Using Equations 4 and 5:

$$R = \frac{190 - 90}{90 - 80} = 10$$

$$X = \frac{90 - 80}{190 - 80} = 0.091$$

A slight misreading of X can make an appreciable difference in F. But by using Equations 4a and 5a we find:

$$R = \frac{90 - 80}{190 - 90} = 0.1$$

$$X = \frac{190 - 90}{190 - 80} = 0.91$$

Now, the correct value of 0.826 is easily read from Fig. 4.

Equations 4a and 5a should only be used with the normal one-shell pass and even number of tube passes. They are inadequate for divided flow or split flow. In this latter type of flow pattern the relationship R = wc/WC is violated.

Sometimes R will be outside the normal range of chart values, and more than six shell passes in series will be needed. In these cases use the following equations, based on work by Bowman:[4]

$$X_{n-2n} = \frac{1 - \left(\frac{1 - RX}{1 - X}\right)^{1/n}}{R - \left(\frac{1 - RX}{1 - X}\right)^{1/n}} \quad (6)$$

$$F_{n-2n} = \frac{\frac{\sqrt{R^2 + 1}}{R - 1} \ln \left(\frac{1 - X_{n-2n}}{1 - R\, X_{n-2n}}\right)}{\ln \left[\frac{(2/X_{n-2n}) - 1 - R + \sqrt{R^2 + 1}}{(2/X_{n-2n}) - 1 - R - \sqrt{R^2 + 1}}\right]} \quad (7)$$

When R = 1, Equations 6 and 7 break down, and Equations 6a and 7a should be used:

$$X_{n-2n} = \frac{X}{1 + nX - X} \quad (6a)$$

$$F_{n-2n} = \frac{\sqrt{2} \left(\frac{X_{n-2n}}{1 - X_{n-2n}}\right)}{\ln \left[\frac{(2/X_{n-2n}) - 1 - R + \sqrt{R^2 + 1}}{(2/X_{n-2n}) - 1 - R - \sqrt{R^2 + 1}}\right]} \quad (7a)$$

Where n = number of shell passes.

Use Equation 6 to calculate X for the desired number of shells in series. Then use this value in Equation 7 to obtain F.

Suppose it is desired to find the MTD correction factor for the following temperature conditions:

Hot	Cold
↓ 410	404 ↑
400	204
10	200

R = 10/200 = 0.05

X = 200/206 = 0.971

Using Equation 6 and three shells in series as an example,

$$X_{3\text{-}6} = \frac{1 - \left(\frac{1 - 0.05 \times 0.971}{1 - 0.971}\right)^{1/3}}{0.05 - \left(\frac{1 - 0.05 \times 0.971}{1 - 0.971}\right)^{1/3}} = 0.70$$

Then, from Equation 7,

$$F_{3\text{-}6} = \frac{\frac{\sqrt{(0.05)^2 + 1}}{0.05 - 1} \ln \left(\frac{1 - 0.70}{1 - 0.05 \times 0.70}\right)}{\ln \left[\frac{(2/0.70) - 1 - 0.05 + \sqrt{(0.05)^2 + 1}}{(2/0.70) - 1 - 0.05 - \sqrt{(0.05)^2 + 1}}\right]} = 0.988$$

To calculate the corrected MTD by computer, block diagraming incorporating Equations 6 and 7 can be found.[5]

Whenever there is a temperature cross in one exchanger, the thermal design should be checked very carefully. (This occurs at a correction factor of approximately 0.8, or when the correction factor is less than 0.8 with shell passes in series.) This should be done for two reasons:

1. It may be more economical to use more shells in series, especially if the units are made of an expensive alloy.

2. The exchanger may not work as well as it was designed for. At close temperature approaches the MTD correction factors tend to break down at low values of F.

Very seldom are three tube passes (or a larger odd number) used. Manufacturing difficulties make every type but a fixed-tube-sheet unit undesirable.

Fischer[6] derived a trial-and-error solution for a one-shell-pass, three-tube-pass unit. One tube pass is in cocurrent flow and the other two are in countercurrent flow. Since more than one-half of the tube passes are in countercurrent flow, the correction factor is higher than in an even-tube-pass unit. A correction-factor chart for one shell is

Two-shell pass with long baffle

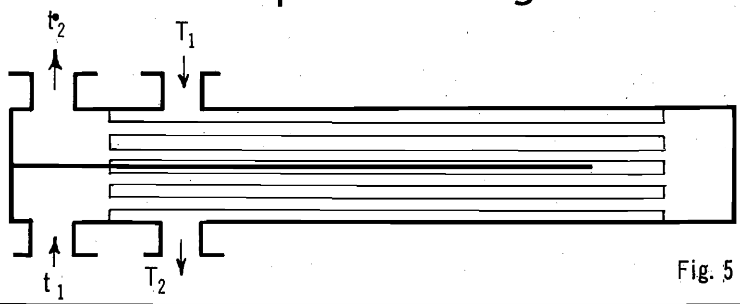


Fig. 5

given. For more than one shell the results are tabulated.

4. Longitudinal baffle

Sometimes when a two-shell correction (or greater) is required, it is possible to use a longitudinal baffle. This flow arrangement is illustrated in Fig. 5.

For four or more tube passes, a 2-4 correction factor is used. This assumes two things:

1. There is a perfect seal between the shell and the long baffle.
2. There is no conduction through the long baffle.

The effect of the second assumption can be ignored with small-shell fluid-cooling ranges. If the cooling range is large and there is a large temperature driving force across the baffle, then baffle conduction will have to be considered. Whister[7] presents an equation for F, for two shell-two tube passes incorporating baffle conduction.

If the two assumptions are valid, and there is a long baffle and two tube passes, no MTD correction factor is necessary because the shell side and the tube side will be in counterflow.

5. Divided-flow, one-tube pass

The divided-flow, single-tube-pass correction factor is seldom used by itself, but is useful to develop correction factors for divided flow-multitube pass and split flow.

In the past there has been some confusion about what is divided flow and what is split flow. This was resolved in the 1959 issue of TEMA.[3] TEMA refers to divided flow as being shell type J. Divided flow is illustrated in Fig. 6.

Frequently the question arises, which of the nozzle arrangements shown in Fig. 6 should be used? The answer lies mainly in what type of impingement protection is used. The two common types are:

1. Removing tubes under the nozzle.
2. A vapor belt or other type of enlarger.

If the first type is used, then Fig. 6a is better. Since there are two smaller nozzles entering rather than one big one, fewer tubes will be removed for impingement protection. Thus, in this arrangement there are more tubes in a given size shell.

If the flow is reversed (flowing up) as in a thermosiphon reboiler, where it is desired to remove tubes for vapor escape area, the same holds true.

If a vapor belt is used, then the nozzle arrangement in Fig. 6b is better. The number of tubes will remain the same in a given shell, no matter what the inlet nozzle size is. The two outlet nozzles will be smaller than the two inlet nozzles shown in Fig. 6a, when condensing downward, and they can be placed closer to the tube sheets. This leaves less space between the nozzle and the tube sheet and a more effective flow pattern is obtained.

A procedure has been developed for constructing a divided-flow, one-tube-pass correction-factor chart,[8] based on a method used by K. A. Gardner.[9]

To develop a chart, an expression for X is used in terms of R and ϕ:

$$X = \frac{\phi^{(2R+1)/2} - 1}{(2R+1)\,\phi^{(2R+1)/2}} + \frac{(1-X)\,(\phi^{(2R-1)/2} - 1)}{(2R-1)\,\phi^{(2R-1)/2}} \qquad (8)$$

Solving for X:

$$X = \frac{(\phi^{(2R-1)/2} - \phi^{-1})\,(2R-1) + (\phi^{(2R-1)/2} - 1)\,(2R+1)}{(2R+1)\,(2R\,\phi^{(2R-1)/2} - 1)} \qquad (9)$$

Divided flow

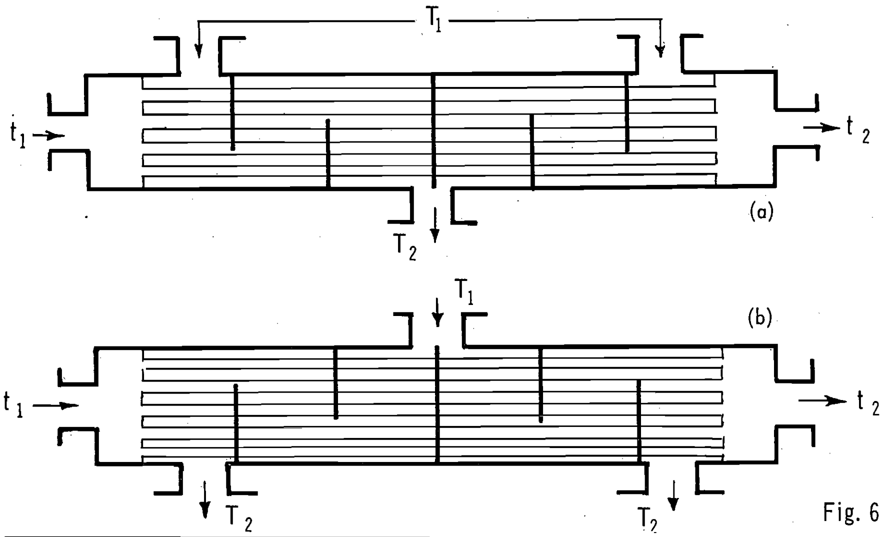


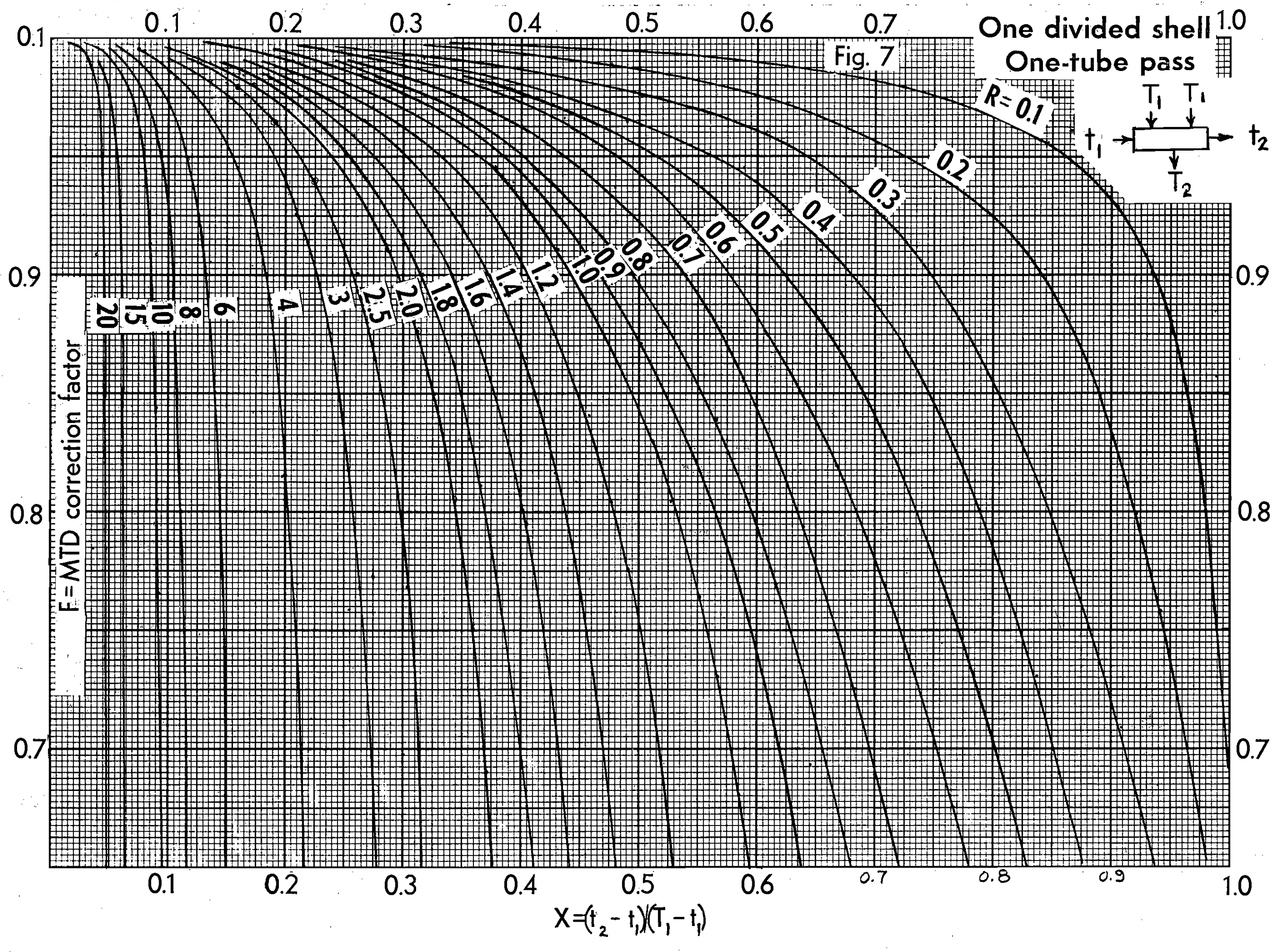
Fig. 6

To calculate F, a value of ϕ is assumed. Then X is calculated from Equation 9. For a given value of R, Equation 10 is used to find F:

$$F = \ln \frac{(1-X)/(1-RX)}{(R-1)\ln\phi} \qquad (10)$$

If it is desired to develop charts for more than one shell pass, a form of Equation 6 can be used. Let n be the number of shell passes in series. Then:

$$X_n = \frac{\left(\frac{RX-1}{X-1}\right)^n - 1}{\left(\frac{RX-1}{X-1}\right)^n - R} \qquad (11)$$



X_n and R are plotted to give the multishell-pass charts.

There are two values of R that require a different calculation procedure. These values are $R = 0.5$, or $R = 1.0$.

When $R = 0.5$, Equation 9 breaks down. Taking the limit of Equation 9 as $(R - 0.5)$ approaches 0, we have:

$$X = \frac{(\phi - 1)/\phi + \ln \phi}{2 + \ln \phi} \quad (12)$$

Equation 12 is used with Equation 10 to give F.

When $R = 1.0$, Equation 9 is still used to calculate X, but Equation 10 will break down when used for F. Taking the limit of Equation 10:

$$F = \frac{X}{(1 - X) \ln \phi} \quad (13)$$

Equation 9 is used with Equation 13 to give F.

When developing charts for more than one shell, Equation 11 will also break down. In that case, use Equation 6a to find the correct value of X for plotting F.

Fig. 7 is an F chart for single-tube pass and one divided shell. **Charts** for the two, three, and four divided shells **may be obtained from the author.**

Developing these charts by hand is quite tedious, but they lend themselves to computer application very well.

A Fortran program is available for calculating F values, **upon request from the author.**

6. Divided-flow, two-tube pass

A trial-and-error solution for F has been presented[8] to use divided-flow, one-tube pass correction:

1. Assume a value for the intermediate temperature between tube passes. A good starting point is:

$$t_b = t_1 + 0.6\,(t_2 - t_1) \quad (14)$$

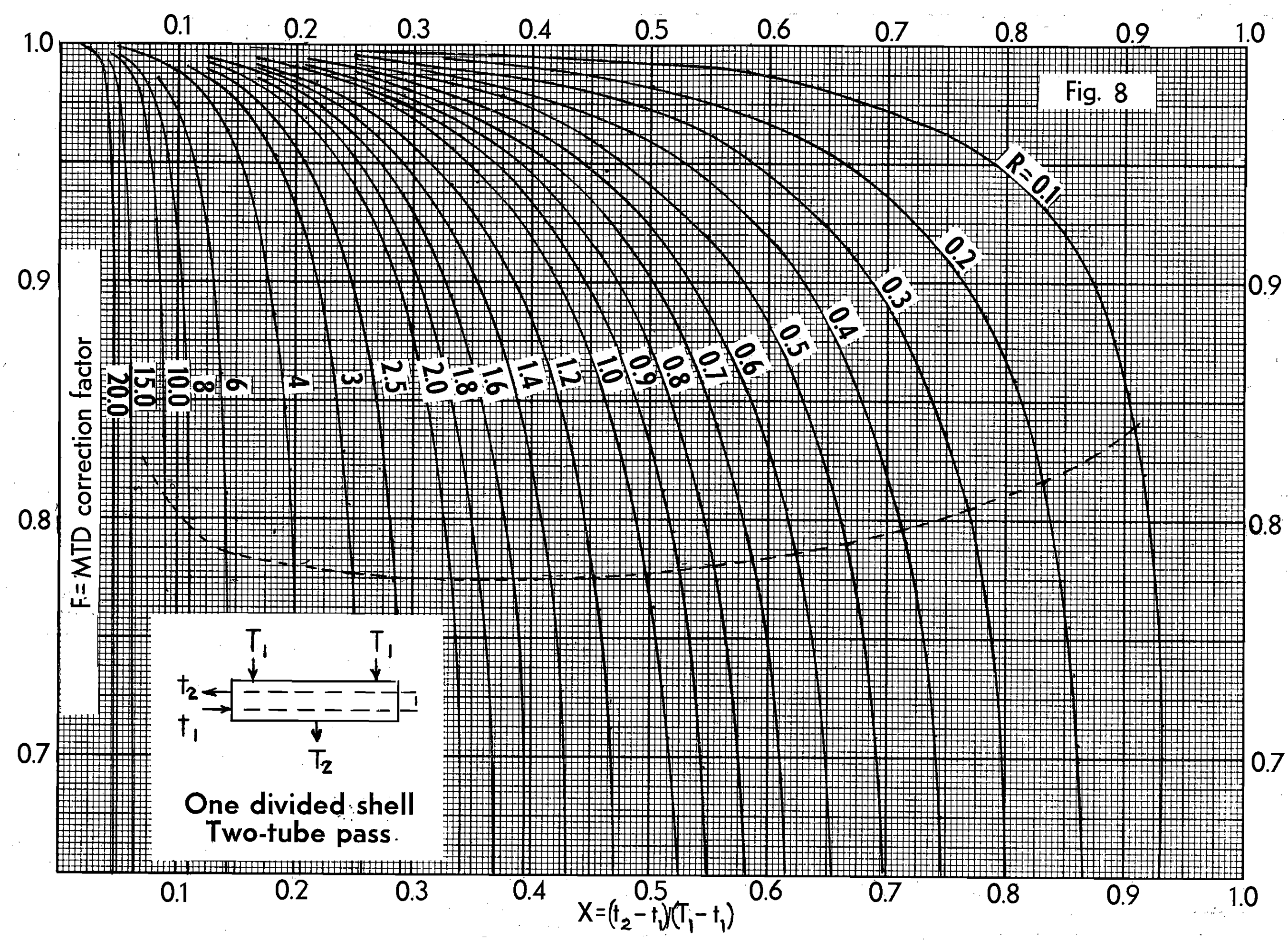
2. Calculate the LMTD in the lower tube pass by using Fig. 7.
3. Calculate the LMTD in the upper tube pass using Fig. 7.
4. Calculate the intermediate temperature (t_b). Since temperature rise in each pass is proportional to its LMTD:

$$t_b = t_1 + \frac{(t_2 - t_1)\,(LMTD)_L}{(LMTD)_L + (LMTD)_V} \quad (15)$$

5. If t_b doesn't check the assumed value, start again using the last value of t_b.
6. Calculate the corrected MTD by weighting the MTD's of the upper and lower tube pass, using Equation 16:

$$\Delta t_m = \frac{t_2 - t_1}{\dfrac{t_b - t_1}{(LMTD)_L} + \dfrac{t_2 - t_b}{(LMTD)_V}} \quad (16)$$

7. The correction factor is backed out by using Equation 3. Example: The shell fluid cools from 200° to 120° F., and the tube fluid heats up from 80° to 120° F.



1. From Equation 14, $t_b = 80 + 0.6\,(40) = 104$.

2. LMTD in the lower tube pass:

Hot fluid		Cold fluid		Difference
200	–	104	=	96
120	–	80	=	40

LMTD = 64

R = 3.33 X = 0.2

$\Delta t_m = \text{LMTD} \times F = 64 \times 0.92 = 58.9$

3. LMTD in the upper tube pass:

Hot fluid		Cold fluid		Difference
200	–	120	=	80
120	–	104	=	16

LMTD = 39.8

R = 5.0 X = 0.167

$\Delta t_m = \text{LMTD} \times F = 39.8 \times 0.882 = 35.1$

4. From Equation 15,

$$t_b = 80 + \frac{(40)\,(58.9)}{58.9 + 35.1} = 105$$

5. Since the calculated value of t_b did not match the value used, use the calculated value and go back to the start.

The final value of t_b is 105.5.

6. With the final value of t_b the MTD's are:

Lower $\Delta t_m = 57$ and upper $\Delta t_m = 32$.

From Equation 16,

$$\Delta t_m = \frac{40}{25.5/57 + 14.5/32} = 44.5$$

7. F = 44.5/57.6 = 0.773

A computer is almost a must to calculate the many different points required for an F chart.

A Fortran program can be written for this type of flow. The biggest problem in putting it on the computer is to calculate the correction factor for divided-flow, one-tube pass. When doing it by hand, as in the above example, use an F chart. Since the F chart was developed by trial and error, we do not have an equation for it. Therein lies the trouble.

Solving Equation 9 for ϕ isn't possible by the usual means. And without ϕ it is impossible to calculate F.

In the computer program developed by the author, ϕ is found by trial and error. To converge on the value of ϕ, use Equation 17 as a first approximation:

$$\phi = \left[\frac{2}{2 - 1.05\,X\,(2R + 1)}\right]^{1/(R+0.5)} \quad (17)$$

After this value is calculated, use the first derivative method of convergence.

Fig. 8 is the F chart for one divided shell and two tube passes. The dashed line extended across the chart shows when the outlet temperature of the hot side is equal to the outlet temperature of the cold side. In comparing this with the normal one-shell pass, the low point is approximately 0.8 while the divided shell is approximately 0.775.

Three F charts for divided-flow, two-tube pass, from two to four

Split flow

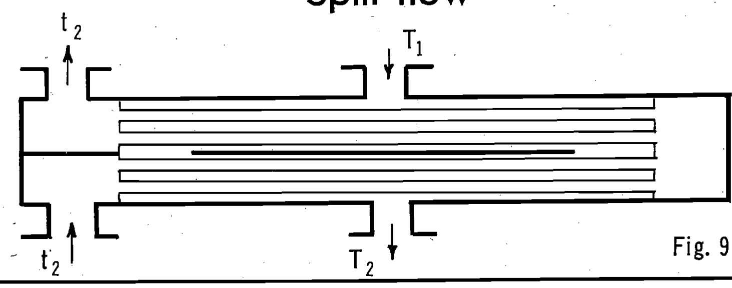


Fig. 9

shells in series, respectively, **may be obtained from the author.**

Split flow. At times, when a high correction factor is desired, a long baffle will give too much pressure drop. In this case split flow can be used. This type of flow is shown in Fig. 9.

Some of the correction-factor charts already developed can be used. By analyzing two divided-flow, one-tube passes in series, it can be seen that it is equivalent to a split-flow-shell, two-tube passes. In like fashion, the four divided-shell, one-tube pass can be used for **two** split-flow shells in series with two tube passes each.

If it is desired to use more split-flow shells in series, use Equation 11. Using the divided-flow correction factors in this manner assumes we have the same perfect conditions listed under longitudinal baffle.

When you have split flow and four or more tube passes, then the previously mentioned charts obtainable from the author can be used, one for a single shell, and one for two shells.

Author Dale L. Gulley lives at 2714 S. 75th E. Ave., Tulsa, Okla.

References

1. Kern, D. Q., "Process Heat Transfer": McGraw-Hill, 1950.

2. McAdams, "Heat Transmission": Third edition, McGraw-Hill, 1951.

3. Standards of TEMA: Fourth edition, 1959.

4. Bowman, R. A., "Mean-Temperature Difference Correction in Multipass Exchangers": Ind. and Engr. Chem., 28, 1936, pp. 541-544.

5. Gulley, D. L., "Use Computers to Select Exchangers": Petroleum Refiner, 39, 1960, pp. 149-156.

6. Fischer, F. K., "Mean-Temperature-Difference Correction in Multipass Exchangers": Ind. and Engr. Chem., 30, 1938, pp. 377-383.

7. Whister, A. M., "Correction for Heat Conduction Through Longitudinal Baffle of Heat Exchanger": Trans. ASME, 69, 1947, pp. 683-685.

8. Gulley, D. L., "Make This Correction Factor Chart to Find Divided Flow Exchanger MTD": Petro/Chem Engineer, July 1962, pp. 143-145.

9. Gardner, K. A., "Mean Temperature Difference in Multipass Exchangers": Ind. and Engr. Chem., 33, 1941, pp. 1495-1500.

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BOOKS

GUIDE DU PETROLE ET DE LA PETROCHIMIE (Petroleum and Petrochemical Guide). 1964 annual edition. Published by Editions O. Lesourd, 252 Faubourg Saint-Honore, Paris 8, France. Price 100 francs. (Franco 104.20 francs.)

The 34th annual edition of this outstanding directory came off the press in Paris in May. It contains more than 1,000 pages of names and addresses of oil, petrochemical, and service companies, plus phone numbers, personnel, company relationships, and other data.

Here, briefly, is an outline of this useful book:

Part 1. Oil. This listing gives details on administrative and professional oil groups, including memberships. It lists French oil companies according to their functions, i.e., exploration and production; transportation and storage; engineering, refining, importing, and distribution; lube oils; and natural gas and products.

Part 2. Petrochemicals and plastic materials. Here you will find information on various French petrochemical companies and organizations, including names and titles of company officials.

Part 3. Oil and petrochemical market. This includes names of companies and other information on firms doing business in the Common Market and the OECD countries, as well as North Africa and the Middle East.

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FEDERAL TAX TREATMENT OF INCOME FROM OIL AND GAS. By Stephen L. McDonald. Brookings Institution, 1775 Massachusetts Ave. N.W., Washington, D.C. 163 pages. Paper $2, cloth $3.50.

This is a summary of a symposium on oil-depletion tax provisions held by a group of professional economists at Brooking Institution. Bulk of the volume is a background paper prepared in advance by Stephen L. McDonald of University of Texas. The remainder is a summary of conference discussion.

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